Energy efficient driving in dynamic environment: Globally optimal MPC-like motion planning framework

Zlatan Ajanović[1], Michael Stolz[1], Martin Horn[2]

Abstract

Predictive motion planning is a key for achieving energy efficient driving, which is one of the major visions of automated driving nowadays. Motion planning is a challenging task, especially in the presence of other dynamic traffic participants. Two main issues have to be addressed. First, for globally optimal driving the entire trip has to be considered at once. Secondly, the movement of other traffic participants is usually not known in advance. Both issues lead to increased computational effort. The length of the prediction horizon is usually large and the problem of unknown future movement of other traffic participants usually requires frequent replanning.

This work proposes a novel motion planning approach for vehicles operating in dynamic environments. The above mentioned problems are addressed by splitting the planning into a strategic planning part and situation dependent replanning part. Strategic planning is done without considering other dynamic participants and is reused later in order to lower the computational effort during replanning phase.

Keywords: eco driving, optimal speed trajectory, dynamic environment, real time capability, replanning

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1 Introduction

Knowledge about the upcoming driving route, the road conditions and the ability to control the vehicle's propulsion is an enabler for optimization of the driving behavior with respect to energy consumption. Discrete dynamic programming (DP) has been used for over a decade now for this purpose (e.g. in research focused on heavy duty vehicles [1] ,[2]). A comparison between different optimization methods (Euler-Lagrange, Pontryagin's Maximum Principle, DP, and Direct Multiple Shooting) was presented in [3]. The work additionally covers an analysis on the DP grid choice, tips on backward and forward dynamic programming, and how to incorporate traffic lights. The authors of [4] showed, that model predictive control (MPC) enables notable fuel savings for vehicles driving on free roads with up and down slopes. Additional usage of MPC was presented in [5] for control of a hybrid vehicle driving over a hill and performing vehicle following. An overview of existing approaches treating this as an optimal control problem and current state of the art can be found in [6].

The integration of traffic lights into optimal motion planning has also been studied intensively. In [7] authors showed an approach for the case of incomplete knowledge about upcoming traffic lights' timing. The case of complete knowledge of the upcoming traffic lights' timing together with Dijkstra's algorithm was studied in [8] and in [9] a MPC based controller was developed with additional constraints imposed from a vehicle in front.

The vehicle following problem is studied in [10]. A possible solution is presented showing different concepts for safe vehicle following, defining helpful concepts such as the safe distance, time-inter-vehicular and time-to-collision. A possible solution for comfort oriented vehicle following is presented in [11] with leading vehicle movement prediction treated as disturbance in an MPC controller. Several publications [12], [13], [14], [15] are approaching planning of optimal overtaking with a different goals. With respect to energy efficiency, all these methods modify an optimal speed trajectory in a way so that it leads to the smallest.

The mentioned publications can be roughly grouped into MPC approaches which execute replanning continuously during driving, and optimal control approaches which plan the entire trip at once. MPC approaches are generally dealing well with dynamic constraints but cannot guarantee globally optimal solutions as their prediction horizon is limited. On the other hand, optimal control approaches generally guarantee globally optimal solutions for the initial problem, but not in presence of disturbances. The proposed approach fills the gap between these two approaches by using optimal trajectory tree and MPC-like replanning scheme.

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2 Problem definition

This work focuses on an energy efficient motion planning algorithm based on dynamic programming in presence of dynamic constraints. Within this computational efficiency of the algorithm is important for achieving online adaptability. A common approach for achieving energy efficient driving is to first formulate an appropriate optimal control problem. This problem is then solved offline and the resulting velocity trajectory is used as a reference for low level speed control.
When applied like this, in real traffic scenarios with other traffic participants or dynamic constraints such as traffic lights, these reference trajectories may not be followed by low level control. This would directly lead to non-optimal driving. To avoid this, the motion of other traffic participants has to be considered as a constraint in the optimization problem as it is shown in [16]. Practically, this is not always possible, as the motion of other traffic participants is only known when they are in a sensor field of view of the ego vehicle, but not when the initial planning is done. Additionally, when predicting future motion of other traffic participants, model uncertainties cause deviations between real and predicted driving over time. To avoid deviations, frequent replanning is necessary. This brings significant computational burdens if the whole trip is considered, which is necessary to achieve a globally optimal solution.

2.1 Optimal control problem

Formally, this problem can be expressed as an optimal control problem with an appropriate cost function. The cost function has to reflect the aforementioned requirement of minimal energy consumption. This includes energy used for propulsion and energy used on-board (e.g., infotainment, component temperature management, air conditioning). A dynamic vehicle model is used to estimate the propulsion force needed to compensate for resistance forces (gravity, air drag, roll resistance) and to provide the required acceleration. Detailed derivation of the model used in this work is presented in [16]. If only energy used for propulsion is considered, energy- efficient behavior would result in smooth, low-speed driving (almost zero). However, as on-board energy usage is proportional to driving time, slow driving increases the overall consumption. The optimal speed trajectory is therefore a balance between these two types of consumption.
In addition, an optimal velocity trajectory has to satisfy several constraints. Constraints can be classified as internal or external. Internal constraints arise from system limitations (e.g. maximum acceleration, velocity, torque), while external constraints are caused by the environment (e.g. traffic signs, other traffic participants). The integration of constraints such as collision avoidance is not straight forward, as these constraints are time and space varying and depend on the driving trajectory of the controlled vehicle itself.

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Although analytical optimization approaches exist, the application to the discussed planning task is problematic, due to incorporating various constraints. The focus therefore in the following work is a numeric optimization, especially graph based approaches, since they are the most flexible and applicable to nonlinear problem.

2.2 Computational complexity

To solve the optimization task numerically, using graph searching methods, state discretization is necessary. By increasing the number of system states considered in the optimization problem, complexity is increased exponentially, as the number of possible state combinations increases exponentially. Each additional state multiplies the number of state combinations by the number of its discretization levels. Additionally, and even more problematic, the number of possible transitions needed to be evaluated each step are increased significantly. Bellman called this problem the *„curse of dimensionality"*.
Other traffic participants represent time and space-varying constraints on both position and velocity. Because of that, travel time must be used as a system state, beside travel distance and velocity. All three system states must be discretized and the whole state space must be searched to achieve a globally optimal solution. This requires significant computational effort.

3 Optimal motion planner

The main idea of the optimal motion planner introduced in this work is based on the combination of the advantages of forward and backward dynamic programming. The planning problem is addressed by splitting into strategic planning and situation dependent replanning. The results once calculated by backward programming, in strategic planning phase, are continuously reused for the ongoing replanning during driving. Replanning is done using forward planning from the actual system state, for a certain prediction horizon into the future, and merged with previously obtained results from backward planning. During replanning, dynamic constraints and additional states (e.g. lanes, travel time) are considered. In this way, the whole trip is taken into consideration along with newly arisen constraints, but only a planning for a defined horizon is executed. This promises benefits of both, forward planning the entire trip (globally optimal solution) and adaptability of MPC with significant reduction in computational effort.

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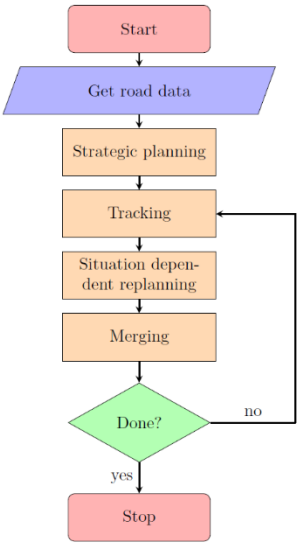


Fig. 1. Optimal motion planner flowchart

3.1 Dynamic programming

Dynamic programming is a preferred method used for solving the optimal control problem discussed in this work. The main advantages are its flexibility and possibility to incorporate different kinds of models and constraints and the fact that it results in a globally optimal solution. It is based on the Principle of Optimality, introduced by R. Bellman [16].

The iterative approach of dynamic programming can be executed starting from the goal state towards the initial state (backward dynamic programming) and vice versa (forward dynamic programming). The advantage of the backward calculation is that the calculated result can be reused during the trip, as it only depends on the final state. This is not the case with the forward calculation, where results are related to specific initial states. On the other hand, the advantage of forward calculation is that other states such as the time of travel can be calculated as the initial time is always known.

3.2 Strategic planning

The strategic planning phase is executed only once at the beginning of a trip, or if the target location changes. It is achieved by using backward dynamic programming starting from a goal state, backward in space. In this phase, only time invari-

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ant constraints are considered (e.g. speed limits) with topological road profile and vehicle model. The results of this phase are *the optimal trajectory tree*, *the cost-to-go map* and *the initial optimal velocity trajectory*. The initial optimal velocity trajectory is one branch of the optimal trajectory tree which passes through the initial state.

Optimal speed trajectory tree

The optimal speed trajectory tree is a tree-like structure formed by connecting all optimal transitions by lines. Together with a cost-to-go map it gives insight into the optimal behavior when only static constraints are present.
It can be noted generally, that if two different trajectories have a common node they will continue on the same trajectory towards the goal. This implies that when planning a trajectory in forward approach, if constraints introduced by other traffic participants are not active any more, a trajectory from a backward planning starting from that state towards the goal can be reused. In this work we will use this property of the optimal trajectory tree to reduce the computational effort needed.
An optimal speed trajectory tree for a problem considered in this work with discretization steps of 5 m for distance and 0.5 m/s for speed is shown on Fig. 2. This map is generated from the goal state towards the start using backward DP. Additionally to the initial optimal trajectory for the given initial condition, multiple other trajectories (branches) for different initial conditions are available.

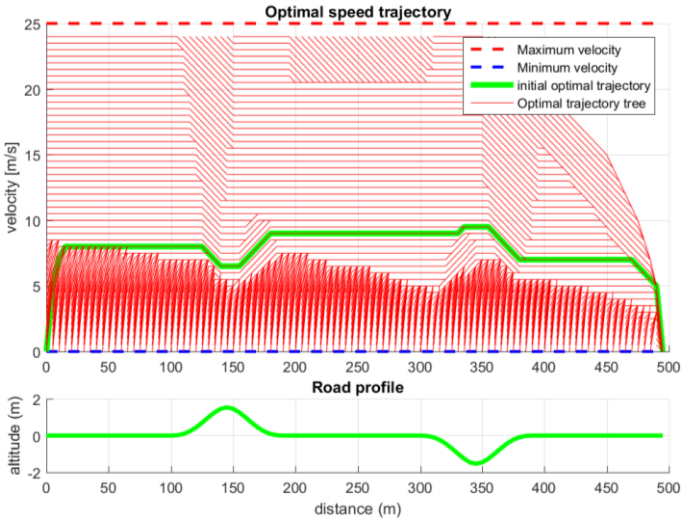


Fig. 2: Optimal speed trajectory tree.

Cost-to-go map

The cost-to-go map provides additional information to the optimal trajectory tree. It represents the minimum energy needed to finish a trip from that state point. It can be achieved by following an optimal trajectory, represented as a branch on the

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optimal trajectory tree starting from that state point. In Fig. 3 the cost-to-go map for the same problem as in Fig. 2 is shown.

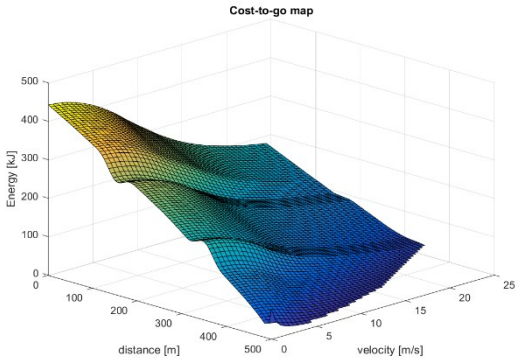


Fig. 3: Cost-to-go map

3.3 Situation dependent replanning

During the replanning phase, the optimal trajectory is adjusted by taking into consideration dynamically arisen constraints. The adjustment is done by replanning the optimal trajectory in an efficient way by reusing the cost-to-go map and an optimal speed trajectory tree. The replanning is done with forward dynamic programming starting from the actual system state in operational space for a defined prediction horizon in the future.

Several safety factors such as maximum time of overtaking execution (constraint on minimum velocity difference), minimum distance from the leading vehicle and clearance needed for lane changing are considered as constraints in this phase.

The future movement of other vehicles is calculated using a simple prediction model that assumes that the leading vehicle will continue moving with constant speed and that it will slow down if it reaches the controlled vehicle (after being overtaken). More sophisticated models of the leading vehicle's velocity which may depend on space, time and the controlled vehicle can be also included.

A principle of operation is shown on Fig. 4. Grey lines represent the optimal trajectory tree, constructed in the strategic planning phase. The blue line is the initial optimal trajectory, which also results from the strategic planning phase. The vehicle drives on the initial optimal trajectory until situation dependent replanning is initiated. By using forward planning, a forward optimal trajectory tree starting from the actual state is constructed (dashed lines) considering dynamic constraints such as other traffic participants, traffic lights, etc.

The merging of two trajectory trees (forward and backward) is done at the end of the forward replanning phase. *Cost-to-come*, the *cost-to-go* equivalent in forward planning, values at the *possible joining nodes* at the end of the replanning horizon

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are summed with *cost-to-go* cost at these nodes from backward planning. In this way, combined costs of moving on trajectories partially planned forward and partially planned backward are calculated. The minimum among these costs is chosen, defining an *optimal joining node*, on the new optimal trajectory (solid black node). Starting from this node backwards, towards the actual state, a *new optimal trajectory* can be constructed iteratively. A *new optimal trajectory* doesn't have to be constructed fully to the final state, if the new replanning will happen while driving within the prediction horizon. In this way, unnecessary calculations can be avoided.

In Fig. 4 no additional system states (e.g. time, lane) are drawn within forward planning, to keep clarity of working principle. Eventually, additional states can be visualized on third dimension in parallel to presented forward planning. During merging, for choosing the optimal joining node the value of these states are neglected and an appropriate node from cost-to-go is chosen based on the velocity and position state values only.

The situation dependent replanning procedure is repeated during the entire trip. It can be triggered by either spatial length traveled, time period, by an event (e.g. detecting of other traffic participants, detecting a significant deviation of predicted motion of other traffic participants) or a combination of these.

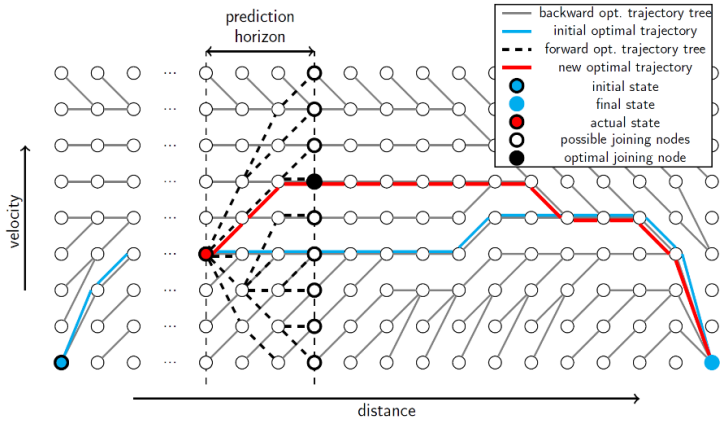


Fig. 4: Situation dependent replanning

3.3.1 Prediction horizon

As spatial discretization is used as a basis for motion planning, the prediction horizon is defined by the length at which a forward planning is executed. It is important to choose a proper horizon length as a so called short sightedness can appear otherwise. Several constraints such as clearance needed for changing a lane can block lane changing if horizon length is not enough. In general, the horizon

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should be as long as computational resources enable it, but keeping in mind that accuracy of motion prediction of other traffic participants decreases with time.
Horizon length depends also on the replanning length as it should be long enough, so that, when next replanning is initiated, enough clearance for lane change is guaranteed (if it exist).

3.3.2 Replanning triggering

The replanning triggering is most likely being determined by a length. This means that the replanning frequency is not constant, as it depends on driving velocity. For real life application this is not a big issue for driving on highway, as there is no big fluctuation of velocity. Frequent replanning is important, if the environment is highly dynamic and the prediction of other traffic participant's motion is not precise. Generally, it is better if replanning length is shorter (replanning frequency higher), but this increases computational effort. Finally a trade-off of these two has to be made.

4 Simulation results

To present advantages of this contribution a realistic driving scenario is simulated. A vehicle is driving on an optimal trajectory on a multilane road and approaches a slower moving vehicle, with an average velocity of 16 m/s with sinusoidal fluctuations of amplitude 1 m/s and period 40s. This un-modeled acceleration introduces a deviation from the predicted motion, as constant velocity model (CVM) is used to predict future motion. The measured actual velocity of other traffic participants, at the time of replanning is used for prediction during planning. In forward planning, the ego vehicle has to drive at least 3 m/s faster to overtake other vehicles.

Table 1. Different motion planning approaches

#	*Replanning length [m]*	*Horizon length [m]*	*Driving time [s]*	*Energy consumed [kJ]*	*Energy cons[%]*
1	***-***	***-***	***309,77***	***2673,1***	***0***
2	***25***	***250***	***305,05***	***2679,9***	***+0.25***
3	***-***	***-***	***320,57***	***2842,3***	***+6,32***

Because of the deviation, the planned trajectory can lead to a collision. Therefore, the ego vehicle is equipped with an ACC, so if the desired trajectory would result in a collision, the ACC would slow down the vehicle. Unfortunately, this causes additional energy consumption as the vehicle is deviating from its initial optimal speed trajectory. Therefore, frequent replanning is necessary to adjust the optimal trajectory to new situations and achieve optimal driving.

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To analyze the potential for energy savings different motion planning approaches are simulated and compared. The results are shown in Table 1. The first approach was initial optimal trajectory without considering obstacles. The second approach is the approach introduced in this work. The third approach is using only initial optimal trajectory and relying on ACC to adapt velocity to avoid collisions.

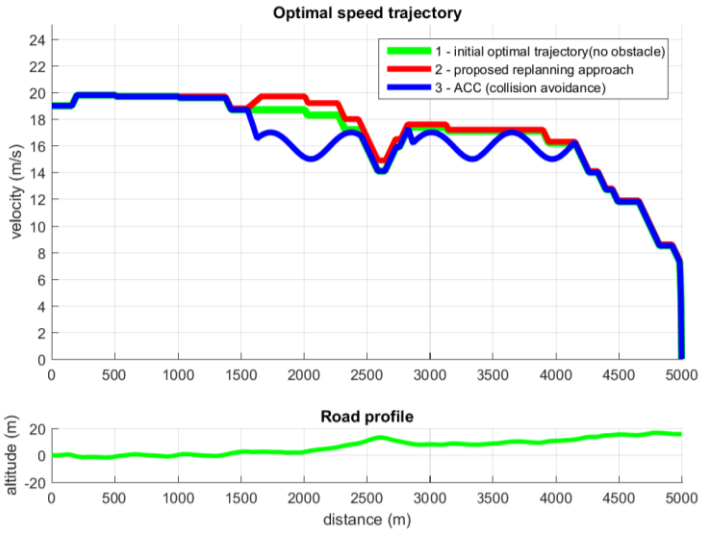


Fig. 5: Simulation results

5 Conclusion

The example reveals that eco-driving algorithms can provide significant energy savings. To be accepted by drivers and to achieve real driving benefits, eco-driving algorithms should be intuitive and adaptable to dynamic environments. They should rely only on the information available onboard at the time of planning. The simulation example showed that using only the initial optimal trajectory planned at the start of a trip, is far from optimal solution in dynamic environments. The presented approach overcomes these problems and presents a novel approach for optimal motion planning in dynamic environments. Reusing cost-to-go map and optimal trajectory tree effectively provides complete trip planning benefits.
It was shown that the proposed approach allows continuous adjustment to dynamic environments, when information is unknown at initial planning. Additionally, problems arising from deviation in prediction of other traffic participant movement are compensated by frequent replanning. Forward planning for the whole trip leads to lowest energy consumption, but this approach is unrealistic as complete

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information is usually not available, and calculation time is significantly longer. Energy consumption in the proposed approach is only negligibly higher.
The novel approach proposed in this work promises an intuitive and adaptable solution for real life eco driving.

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